Table 2 Estimated parametrs

	Reduced finite-element model	Correcting term	Corrected model
	$\times 10^5$		
k_{11}	0.28351	−0.01343	0.27008
k_{21}	0.27119	0.02859	−0.24259
k_{22}	0.39445	−0.04031	0.35414
k_{31}	0.11094	−0.02686	0.08408
k_{32}	−0.27119	0.02859	−0.24259
k_{33}	0.28351	−0.01343	0.27008
	$\times 10^{-1}$		
m_{11}	5.8509	0.01424	5.8651
m_{21}	0.47902	0.02570	0.50472
m_{22}	5.5981	0.04427	5.6423
m_{31}	−0.25279	0.02848	−0.22432
m_{32}	0.47902	0.02570	0.50472
m_{33}	5.8509	0.01424	5.8651

third eigenvalues. In the computation of the second eigenvalue, the reduced model is marginally better than the corrected model. Overall, the corrected model is superior to the reduced model in replicating the measured acceleration spectra. The estimated parameters deviate minimally from the parameters of the initial model and may be considered to be physically meaningful in the sense that a finite-element model is meaningful. As a final test of goodness, the model is able to predict accurately the response of the beam to loading conditions which differ from the loads applied in the test.

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Review of Composite Rotor Blade Modeling

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Introduction

HELICOPTER rotor blades are typically built-up, composite structures and made of materials that may be anisotropic and nonhomogeneous. The initial impetus for the use of composites was the very significant improvement in fatigue life and damage tolerance of the blades and later the benefits afforded by the ability to incorporate more refined aerodynamic design into planform and airfoil section geometries. In recent years, the subject of aeroelastic tailoring has received attention (see Ref. 1). For advanced rotor blades, composite materials provide opportunities for structural simplicity of hingeless and bearingless designs and structural couplings to improve the aeroelastic stability and response of these configurations (see Ref. 2).

Most previously developed structural models have been limited to isotropic material properties. In modern rotor blades and flex beams, there may be coupling between extension, bending, torsion and shear deformation; warping effects may be much more significant. These complexities generally invalidate the Euler-Bernoulli beam assumptions that plane cross sections remain plane and perpendicular to the elastic axis. This paper is concerned with reviewing the modeling of such composite blades having arbitrary cross sections. A structural theory that is sufficiently general to treat such blades, with their varieties of cross sections, spanwise nonuniformities, and potentially large deflections, does not yet exist.

Research has yielded significant advances over the years in our ability to analyze beams with *simple* cross-sectional geometries such as circular tubes, channels, I-beams, rectangular boxes, cruciforms, etc. In particular, certain theoretical results have been systematically and thoroughly validated by correlation of experimental data with numerical simulation, even down to the prediction of strain at points.[3] However, successful analysis of these simpler structures does not necessarily imply that application of the same analyses to general, composite rotor blades will also be successful. The composite rotor-blade problem is of enormous complexity so that a "frontal assault" in which all the power of continuum mechanics and analysis is rigorously brought to bear may be impractical due to the computational effort that would be required. Certainly three-dimensional, finite-element methods such as the method in Ref. 4 could be employed, but this "brute force" technique is quite expensive, and the form of the results is not amendable to easy interpretation. Since rotor blades generally are much longer than their lateral dimensions, a one-dimensional model would seem feasible, a least from a computational point of view.

Previous Work

Although one-dimensional (i.e., beam) kinematics can be formulated in a rather elegant fashion,[5,6] constitutive laws in terms of known, three-dimensional elastic constants for small strains can only be approximations if the structure is to be treated as one-dimensional. Here the term "constitutive law" refers to the broad class of relations between generalized stress and strain which, for beam problems, entails knowledge of the shear center, various "warp" functions for both in-and out-of-plane cross-section deformation, and possibly as many as several dozen "stiffnesses" derived from various integrals over the cross section of the beam weighted by material moduli and various powers of cross-section coordinates and warp functions and their derivatives. Here and throughout the paper the term warp is used to denote not only in the usual sense of out-of-plane cross-sectional deformation, but also in the more general sense of in-plane, cross-sectional deformation.

In Refs. 6–8, a kinematical basis for beam theory is derived based on the concept of decomposition of the rotation tensor. The beam cross section is postulated to displace and rotate as a rigid body without explicit restriction on the magnitude of the motion. Further deformation of the cross section can be characterized by quantities whose magnitude is small in some sense. In particular, the total rotation at any point in the beam is represented as 1) a large global rotation of the reference triad, which is associated with a frame whose displacement

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and rotation capture all rigid-body motion of the reference cross section; 2) a small rotation due to shear which is constant over the cross section but which does *not* represent a rigid-body rotation of the reference cross section; and 3) a local rotation whose magnitude may be small to moderate, which varies over a given cross section and which vanishes at the reference axis. Since all cross-sectional deformation in and out of the cross-sectional reference plane is in some sense small, there is at least the possibility that the warp functions could be determined on the basis of linear theory. This was the conclusion of Parker[9] from an asymptotic analysis with slenderness ratio as the small parameter.

The general kinematical framework of Danielson and Hodges[6] requires only two assumptions: 1) that the strain components are small relative to unity and 2) either that the maximum magnitude of the local rotation is of the order of the square root of the maximum strain for *moderate* local rotation or of the order of the strain for *small* local rotation. A complete theory based on this kinematical formulation has not yet been developed. For composite rotor-blade analysis, shear deformation and warp should be incorporated along with initial curvature and twist, as in the example of Ref. 6. Incorporation of general applied and inertial forces does not appear to be a formidable obstacle. However, in order to be a practical tool for rotor-blade analysis, a modeling approach is needed for determination of the structural operator which includes taking into account anisotropic materials and the built-up nature of the blade.

Work in this area can be classed into two distinct areas: 1) the use of a specialized, simple model for the blade cross section in order to assess the stability of rotor blades for various values of ply orientation and other geometric parameters and 2) the development of modeling approaches so that the three-dimensional constitutive law from general, anisotropic elasticity can be reduced to a simple one-dimensional form for the beam problem.

The first category is not the subject of this review and will only be briefly summarized. This work has been chiefly done by Hong and Chopra,[10,11] who developed a beam finite-element analysis for flap-lag-torsion stability of hingeless and bearingless rotor blades in hover in which the blade was treated as a single-cell beam composed of an arbitrary layup of composite plies. Stiffness coupling terms caused by bending-twist and extension-twist couplings were correlated with different composite ply layups. The results show that such couplings can have significant effect on the stability and serve as an impetus for further development of analytical capability.

Work in the second category is the main concern of this paper and focuses on the determination of the shear center location, warp function(s), and cross-section properties of arbitrary rotor-blade cross sections. One especially nice feature common to most of the work described in this section is that the cross-sectional portion of the analysis is based on a linear, two-dimensional approximation. Thus, this analysis can be done once for each cross section and is independent of nonlinear global deformation. This partial decoupling in published work is usually assumed to be possible without rigorous proof.

For the modeling approach associated with published work on composite beams, there are two separate approaches which have been developed practically independently of one another. One might be termed an analytical approach, and the other a finite-element-based approach.

Analytical Approaches

The analytical approach is characterized by calculation of the warp function, shear-center location, and stiffness properties in closed form or by simple analytical approximations. The approximations include such things as substitution of a simple, cross-sectional geometry for the actual one being analyzed and the use of low-order polynomials for the form of the warping displacements.

Hegemier and Nair[12] developed an analysis for heterogeneous, transversely isotropic elastic beams with built-in twist. Although the strain-displacement relations are insufficiently general for all helicopter blade applications, there are three noteworthy and relevant contributions of this paper for present purposes. First, a St. Venant solution is presented for free torsion of a composite beam with a heterogeneous cross section that is made up of a collection of homogeneous regions. Appropriate jump conditions to be satisfied by the warp function at inter-region boundaries are given along with formulas to allow determination of the shear center location. Second, the shear-center location is assumed to be slowly varying along the beam reference axis and is determined from a *linear* solution for the shear stress distribution in a transversely loaded uniform cantilever beam with a cross section identical to the one at which the shear center location is desired. This is equivalent to an assumption that the distribution of shear stress over the cross section is similar for all cross sections along the beam. Third, a relatively general material law with five independent elastic constants for transversely isotropic materials is simplified by the assumptions that the beam undergoes uniaxial stress and that the plane normal to the elastic line is the plane of isotropy. This leaves only two independent constants, E and G, in the strain-energy function.

Mansfield and Sobey[13] derived expressions for the coupled torsional, extensional, and flexural stiffnesses of a simplified helicopter blade model consisting of a hollow composite tube. Mansfield[14] extended the theory to two-celled beams. As pointed out by Rehfield,[15] these theoretical developments are rather difficult to follow, and a single, variationally consistent theory does not clearly emerge. Although transverse shear and warping of the beam cross section are now known to be significant in modeling composite beams, neither is included in these works. However, the authors expressed several innovative ideas about aeroelastic tailoring that are only now beginning to be explored. The intent of the analysis was primarily to give the analyst insight to create desired couplings rather than to just analyze a given structure.

A similar approach was developed by Rehfield[15] in which a general rotor-blade cross section is approximated as a single-celled box beam whose torsional warping function can be determined analytically. Subsequent unpublished work allows for multi-celled blades and utilizes the unit load theorem to determine the shear center. As with Mansfield and Sobey, Rehfield's primary intent is to give the analyst sufficient understanding of the couplings and mechanisms to foster creation of desirable coupling effects. The theory is cast in terms of a total potential energy formulation, and explicit formulae for all the elements of the stiffness matrix are given. The main difference between this model and those of Mansfield and Sobey[13] and Mansfield[14] is the inclusion of restrained torsional warping and transverse shear deformation. Nixon[2] presents correlation of experimental data with Rehfield's analysis which shows that transverse shear is extremely important. Hodges, Nixon and Rehfield[15] further demonstrated the efficacy of Rehfield's[16] approach finding favorable correlation between this relatively simple theory and a NASTRAN-finite element model for a beam with a single-closed cell. Other deformation modes, such as extensional warping and flexural warping identified by Rehfield and Murthy,[17] are not considered in the more recent work.

Finite-Element-Based Approaches

The finite-element method offers a versatility and modeling flexibility that no analytical method can match. It allows one to determine the warp functions, shear center, and elastic properties for any general cross-section geometry that can be modeled with standard two-dimensional finite elements. It is frequently argued, however, that along with this power comes a loss of physical insight.

Wörndle[18] presented a method for determination of the shear center and warping functions based on a two-dimensional, finite-element analysis. The analysis is restricted to transversely isotropic materials; here the material fibers must run uniaxially. This excludes, for all intents and purposes, use of this analysis for aeroelastic tailoring applications since the essence of most tailoring schemes involves orientation of material fibers in directions other than along the blade. The analysis of a nonuniform beam is actually carried out in terms of several uniform cantilever beams, each with a cross section that is the same as one at each of several stations of the nonuniform beam being analyzed. A finite element model of the cross section yields the out-of-plane warping function and the shear-center location. Although the out-of-plane warping is determined for both torsion and shear strain only the torsional warping is used in the global deformation analyis. It is further assumed that material and geometric properties of the blade are slowly varying and that the stress field is uniaxial. The analysis is reported to be two orders of magnitude faster than a three-dimensional, finite-element analysis of a uniform beam.

Bauchau[19] developed an anisotropic beam theory in which out-of-plane cross section warping is expanded in terms of so-called eigenwarpings. Other types of warping, such as that due to in-plane deformation, are neglected, and the analysis is restricted to closed, multicelled, thin-walled beams. Rather than assuming a uniaxial stress field, Bauchau assumes that the cross section is rigid in its own plane. Although the analysis is restricted to the transversely isotropic case, it was extended by Bauchau, Coffenberry, and Rehfield[3] to include general orthotropy. The eigenwarpings are determined from the solution of a eigenproblem over the cross section. A truncated set of the eigenwarping functions determined from a two-dimensional, finite-element solution, can be used to enhance the displacement or stress fields based on either a displacement-based minimum potential energy or on a mixed Reissner variational principle, respectively. In practice, only a few eigenwarpings are needed, as shown in Ref. 3, where only torsional warping and shear deformation are included. As described by Bauchau and Hong,[20] the analysis, complete with graphical interface, requires only a few minutes on an AT-class personal computer to determine the shear-center location and the stiffness properties of the cross section. Furthermore, Ref. 20 reports a substantial savings in computational effort if only torsional warping and transverse shear are considered in the nonlinear analysis vs a more general analysis with the eigenwarpings as done by Bauchau and Hong.[21]

Kosmatka[22] removed the restriction to transversely isotropic materials in Wörndle's analysis to include blades made of orthotropic materials with arbitrary fiber direction. The analysis was primarily developed as a method for analyzing highly swept, curved blades (propfan configurations) constructed of anisotropic composite materials but is applicable to rotor blades as well. Kosmatka's treatment of nonuniformities is the same as Wörndle's and also resulted in a two-dimensional, finite-element analysis by which the St. Venant solution for an axially uniform, nonhomogeneous, anisotropic beam is determined. Rather than considering beam cross sections that are rigid in their own planes or beams with uniaxial stress fields as in previously described work, Kosmatka uses the complete strain energy of the beam in determining the in- and out-of-plane warping. For an anisotropic beam, equations governing the in- and out-of-plane warping functions are fully coupled. In his nonlinear analysis however, only the torsional warping is used, and the stress field is restricted to be uniaxial. The numerical analysis of a cross section runs on a minicomputer in a few seconds and on a desktop computer in a few minutes, at least an order of magnitude less time consuming than a linear, three-dimensional analysis of a uniform beam.

Giavotto et al.[23] formulated a two-dimensional, finite-element-based procedure for determining generalized warping functions, the shear center and cross-sectional properties. Though the formulation is similar to those described above, differences there are also some significant differences. First, the analysis is valid for general anisotropy and includes both in-and out-of-plane warping. Second, the complete strain energy is used instead of approximations of uniaxial stress or cross-section in-plane rigidity. Finally the two-dimensional, finite-element, cross-sectional analysis reduces to a linear system of second-order differential equations with constant coefficients, which possesses both general eigensolutions (exponential in axial coordinate and called transitional solutions in the paper) and particular solutions (linear in axial coordinate and called central solutions in the paper). The particular solutions correspond to the warping displacements of the beam due to six independent stress resultants (three forces and three moments as in Kosmatka's formulation). The eigenvectors of the eigensolution are interpreted as the warping displacements due to end effects with the associated eigenvalue supplying information regarding the decay of that mode as one moves away from the ends. Thus in terms of cross-sectional analysis, this work is the most general of all described so far. Having been expressed mostly in terms of matrices, however, the formulation is no less tractable than the others. Good correlation of the finite-element code based on this work with analytical results is demonstrated.

The work of Giavotto et al[23] was extended by Borri and Mantegazza[24] and Borri and Merlini[25] to include nonlinear deformation. Similar to Refs. 20 and 21, the linear, two-dimensional portion of the analysis can be done once per cross section and independently of the nonlinear analysis. The properties thus obtained are then used in the nonlinear analysis of the global deformation. The nonlinear global deformation analysis contains generalized warping displacements from the particular solution; these warping displacements are expressed in terms of the global deformation variables and do not result in additional kinematical variables. It is also possible to include extra kinematical variables in the nonlinear analysis from the eigenvectors associated with the exponential solutions of the linear eigenvalue problem. Elements based on these "transitional" solutions would probably be needed only near ends of the blade in order to account for the expanded decay length normally associated with composite blades. The main unverified assumptions in this work are that 1) a linear two-dimensional analysis of a beam cross section is sufficient to determine the elastic properties of a beam for use in a separate nonlinear global deformation analysis and 2) a nonuniform beam can be analyzed by considering several of its representative cross sections in separate, two-dimensional analyses. Presumably if the cross section dimensions are not varying too rapidly (just how rapidly is not known), the latter assumption will not cause the accuracy of the analysis to suffer. Published information concerning the number of and which warping functions are necessary for analysis of particular beams is not yet available.

Lee and Stemple,[26] and Stemple and Lee[27] developed a finite-element-based approach in which the warping behavior is determined through specification of warping nodes over the cross section. These nodes along with regular beam, finite-element nodes, allow for a unified treatment of the bending, torsional and warping deformation of nonuniform, composite rotor blades. The published work considers only thin-wall cross sections, and out-of-plane warping. This approach requires more computing effort in comparison with analyses of the type of Wörndle,[18] Bauchau,[19] and Borri et al.[24,25] in which the complex behavior due to cross-sectional deformation is separated from the nonlinear beam analysis.

Evaluation and Recommendations

Both Rehfield's[15] and Bauchau's[19] methods, analytical and finite-element-based methods, respectively, yield results of comparable accuracy for box beams according to experimental work reported in Ref. 3. Neither of these methods has yet been developed and validated to the degree necessary for general-purpose analysis of composite rotor-blade or flex-beam cross sections.

The finite-element-based work is sometimes criticized as being too complicated. However, it should be noted that the finite-element procedures described by Wörndle,[18] Bauchau,[19] Giavotto et al.[23], and Kosmatka[22] have certain advantages. The cross-sectional, finite-element analysis is linear and two-dimensional; it has to be done but once for any given cross-section geometry and material layout; it can be programmed with an easy-to-use graphical user interface; and it is very efficient requiring at most a few minutes on typical desktop microcomputers. Hence, it is the opinion of the author that this and other such analyses should be evaluated based on their accuracy in determining the warp functions(s). Once these quantities are known, then the stiffness properties and shear center of the blade cross section can be calculated. In other words, this approach should not be rejected because of complexity, per se, especially if the complex part of the analysis can be so easily computerized and hidden from the user. As pointed out above, some of these analyses are much more general than others; whether this generality is necessary is unknown at this point. For example, the overall importance of in-plane warping deformations and associated transverse normal and distortion shear strains, as found in work of Ref. 23, is not known.

On the other hand, the analytical approaches have their place as well. The finite element methods are less likely to give the analyst the kind of familiarity and understanding of the problem to allow him to intentionally create certain types of couplings. The analytical approaches thus could play a significant role in preliminary design, and the finite-element approaches could, along with formal optimization allow for refinement in the detailed design stage.

Several questions remain in formulating a general composite, rotor-blade analysis impacting both kinematics and constitutive laws. The overall question is whether a one-dimensional beam theory with appropriate kinematics and material constants for composite rotor blades, can be made sufficiently accurate for general composite, rotor-blade analysis. Contributing to this question are several subordinate questions.

1) How far must one go in defining in- and out-of-plane cross-sectional deformation? Are simple analytical expressions adeqate or does one need a more precise determination such as from finite elements? Does one need all the warping displacements or will only the torsional warping suffice?

2) To what degree can cross-sectional deformation be ignored in calculation of the inertial and applied load terms?

3) How important are the types of solutions identified in Ref. 23, the particular solutions which are linear with axial coordinate vs the exponentially varying solutions? One could speculate that the importance of an exponentially varying mode in a nonlinear global, finite-element analysis should depend on the magnitude of the decay length for the blade and how close the element is to a point along the blade where warping is restrained.

4) How accurate is the use of cross-section properties resulting from a two-dimensional, finite element analysis of several, distinct cross sections of a nonuniform beam? How large would the nonuniformities have to be in order to invalidate this kind of appoximation?

5) To what degree can approximate determination of the various warp functions for cross-sectional deformation (usually cast as a linear problem) be decoupled from the nonlinear aspects of the kinematics?

These questions can be answered by attempting to validate various theories by correlation of their results with experimental data. The experimental specimens must include not only beams with simple geometries and distribution of materials, but also realistic rotor-blade sections. It may be that some theories are only useful for static problems, and others will accurately treat both static and dynamic problems. Similarly, it may be that some theories will handle only beams with closed cross sections and others will work for beams with more general cross sections.

Concerning item 5), it may be possible to increase one's confidence that such decoupling is true, at least with "engineering judgment" arguments that are based on the restriction of the analysis to small strains. Massive bodies in which strain components are small can only be subject to small *elastic* displacements and rotations. On the other hand, thin bodies such as beams and plates can undergo finite elastic displacements and rotations even with small strains. Considering a beam as a collection of cross sections or "segments" of very small length, one can determine that the global deformation of a beam is governed by nonlinear equations even when the strain is small. The nonlinearities are kinematical in nature and are due to finite, rigid-body displacement, and rotation of these cross sections. In fact, one can identify a frame associated with the reference cross section as discussed by Simo and Vu-Quoc[5] and Danielson and Hodges.[8] The displacement and rotation of this "intrinsic frame" are not necessarily small, even if the strain components are small, because a beam's lateral dimensions are in some sense small relative to its overall length. However, all other deformation can be characterized as displacement of material points and rotation of material elements relative to the intrinsic frame. Thus, the *deformation* of the reference cross section (as distinct from is rigid-body displacement and rotation) can be assumed to be small in some sense.

Let the reference cross section be represented, for discussion purposes, as a thin, flat plate. It can experience large, rigid-body displacement and rotation, but its deformation can be examined independently of its rigid-body motion. Hodges[7] and Danielson and Hodges[6] have already shown that local rotation (i.e., rotation of material elements in the cross section relative to the intrinsic frame) of the material elements in the reference cross section may be regarded as either small or moderate. For the small local rotation case, the situation is easy to assess. Considering this deformation analogous to that of a thin plate with *small* bending rotations, one can infer that all components of displacement (whether in- or out-of-plane) for point in the cross section must be of the order of the strain. For the moderate local rotation case, matters are somewhat more complicated. Considering this deformation to be analogous to that of a thin plate with *moderate* bending rotations one can infer that the in-plane displacement of a point in the cross section must be of the order of the strain. However, out-of-plane deformation of the reference cross section must be regarded as of the order of strain to the one-half power. This leaves open the possibility of kinematical nonlinearities in terms of at least the out-of-plane warping displacement.

With these observations, it may be possible to establish limits on the applicability of the linear, two-dimensional analysis for obtaining the elastic properties of a beam cross section. Ignoring body forces, one can derive a two-dimensional, variational principle, which governs both in- and out-of-plane warping. Then, conceptually, the warping can be obtained from the solution of the resulting *nonlinear* equations. A set of such nonlinear equations has been obtained in ongoing preliminary work by the author and one of his coworkers. The linear terms of these equations agree with those of Ref. 23; studies are underway to assess the importance of the nonlinear terms in these equations. The character of the nonlinear terms depends on whether one regards the local rotation as small or moderate. In either case, the beam's transverse normal strains and distortion shear strain correspond to in-plane stretching and shearing of the reference cross section, respectively. These strain components, not considered in most work described in this paper, do not appear to be negligible.

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Minimum Weight Design of Rotorcraft Blades with Multiple Frequency and Stress Constraints

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Introduction

AN important consideration in helicopter rotor blade design is to reduce vibration without increasing blade weight. Rotor blade vibration can be reduced by separating its natural frequencies from the harmonics of the airloads or the excitation frequencies to avoid resonance. In the conventional design process this is usually done by post-design addition of nonstructural masses, which often leads to weight penalties. Today, one of the more promising design approaches is the application of optimization techniques during the design process to reduce vibration.[1,5] Some recent work is devoted to reducing vibration of modal shaping[1] or by controlling the vertical hub shears and moments.[2] An early attempt at optimum blade design for proper placement of natural frequencies is due to Peters,[3] who addresses the optimum design of a rectangular blade with frequency and autorotational inertia constraint. Frequency placement alone was also addressed in Ref. 4 using the optimality criteria approach. In the problem addressed in this Note, blade weight is the objective function and constraints are imposed on natural frequencies, autorotational inertia, and centrifugal stress. This is an extension of the work of Ref. 5, where only the frequencies of first flapping dominated and first lead-lag dominanted blade modes were constrained, without any constraint on blade stress.

Blade Model

The blade model includes a nonuniform box beam located inside the airfoil. Its total weight W has two components, W_b and W_o, where W_b denotes the box beam weight and W_o represents the nonstructural weight of the blade, including the weight of the skin, honeycomb, etc., along with the weight of the tuning masses added to the blade. The blade is discretized into finite segments and the blade weight in discretized form is given as

$$W = \sum_{J=1}^{N} \rho_j A_j L_j + \sum_{J=1}^{N} W_{o_j} \tag{1}$$

where N denotes the total number of segments and p_j, A_j, L_j, and W_{o_j} denote the density, the cross sectional area, the length, and the nonstructural weight of the jth segment, respectively. The autorotational inertia (AI) of the blade is calculated as

$$AI = \sum_{J=1}^{N} W_j r_j^2 \tag{2}$$

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